Answer Key

SECTION I, Part A

1. B	2. C	3. E	4. C	5. D	6. E	7. D	8. A	9. C	10. D
11. B	12. A	13. E	14. C	15. B	16. D	17. C	18. A	19. D	20. E
21. C	22. B	23. D	24. D	25. E	26. B	27. C	28. B		

SECTION I, Part B

76. D	77. A	78. C	79. D	80. A	81. D	82. C	83. E	84. D	85. B
86. D	87. A	88. D	89. D	90. B	91. C	92. A			

SECTION I, Part A

1. B $\lim\limits_{x\to 1} \dfrac{x-f(x)}{\left[g(x)\right]^2+1} = \dfrac{\lim\limits_{x\to 1}\left[x-f(x)\right]}{\lim\limits_{x\to 1}\left[g(x)\right]^2+1} = \dfrac{1-2}{\left[-1\right]^2+1} = -\dfrac{1}{2}$

2. C $f(x)=2^{\tan x}$, $f'(x)=\ln(2)\cdot 2^{\tan x}\cdot \dfrac{d}{dx}(\tan x)=\ln(2)\cdot 2^{\tan x}\cdot \sec^2 x$

$f'(\frac{\pi}{4})=\ln(2)\cdot 2^{\tan(\frac{\pi}{4})}\cdot \sec^2(\frac{\pi}{4})=\ln(2)\cdot 2\cdot(\sqrt{2})^2=4\ln 2=\ln 2^4=\ln 16$

3. E If f is continuous on $(-\infty,\infty)$, then $f(2)=\lim\limits_{x\to 2^+} f(x)$.

$\Rightarrow\ 2a-3=\lim\limits_{x\to 2^+}(x^2+a)\ \Rightarrow\ 2a-3=2^2+a$

$\Rightarrow\ a=7$

4. C $\Delta x=\dfrac{b-a}{n}=\dfrac{2-1}{4}=\dfrac{1}{4}$

$\int_1^2 e^{1/x}\,dx=\dfrac{1/4}{2}\left[f(1)+2f(1.25)+2f(1.5)+2f(1.75)+f(2)\right]$

$=\dfrac{1}{8}(e+2e^{1/1.25}+2e^{1/1.5}+2e^{1/1.75}+e^{1/2})$

5. D $\dfrac{d}{dx}\left[\dfrac{1}{3}\sec^3 x-\sec x+3\right]=\dfrac{1}{3}\cdot 3\sec^2 x\cdot\dfrac{d}{dx}(\sec x)-\sec x\tan x=\sec^3 x\tan x-\sec x\tan x$

$=\sec x\tan x(\sec^2 x-1)=\sec x\tan x(\tan^2 x)$

$=\tan^3 x\sec x$

6. E The question is asking for the convergence of the <u>sequences.</u>

I. $\lim\limits_{n\to\infty} \dfrac{4n}{3n+2} = \dfrac{4}{3}$. The sequence converges.

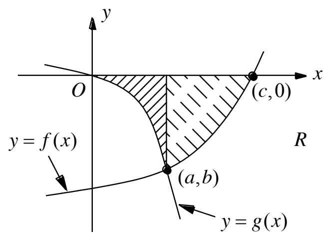
II. $\dfrac{2^n - 9}{e^n} < \dfrac{2^n}{e^n} = (\dfrac{2}{e})^n$. $\lim\limits_{n\to\infty} (\dfrac{2}{e})^n = 0$ because $\dfrac{2}{e} < 1$. The sequence converges.

III. $\lim\limits_{n\to\infty}\left[n\sin(\dfrac{1}{n})\right] = \lim\limits_{n\to\infty}\left[\dfrac{\sin(1/n)}{1/n}\right] = \lim\limits_{t\to 0^+}\left[\dfrac{\sin(t)}{t}\right] = 1$. (Substitute $t = \dfrac{1}{n}$.)

The sequence converges.

7. D The volume of the solid obtained by revolving R about the x-axis is

$$\overbrace{\pi\int_0^a [g(x)]^2\,dx}^{\text{Volume of the solid obtained by revolving } g(x) \text{ about the } x\text{-axis from } x=0 \text{ to } x=a.} + \overbrace{\pi\int_a^c [f(x)]^2\,dx}^{\text{Volume of the solid obtained by revolving } f(x) \text{ about the } x\text{-axis from } x=a \text{ to } x=c.}.$$



8. A $\lim\limits_{h\to 0}\dfrac{f(h)-1}{h} = \lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{h}$ Given $f(0)=1$.

$= \lim\limits_{h\to 0}\dfrac{f(0+h)-f(0)}{h} = f'(0)$ $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h} = f'(a)$

$= -1$ Given $f'(0) = -1$.

9. C The length of a curve from $x=a$ to $x=b$ is $L = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$.

We can conclude that $f'(x) = \sin(2x) \Rightarrow f(x) = -\dfrac{1}{2}\cos(2x)$.

10. D The expression $\lim\limits_{n\to\infty}\dfrac{1}{n}\left[\left(\dfrac{1}{n}\right)^3 + \left(\dfrac{2}{n}\right)^3 + \ldots \left(\dfrac{3n}{n}\right)^3\right]$ is a Riemann sum for the function $f(x) = x^3$

with $\Delta x = \dfrac{1}{n}$ on the interval $[0,3]$. The limit is equal to $\int_0^3 x^3\,dx$.

11. B $\sum\limits_{n=1}^{\infty} e^{-n+1}\cdot 2^n = \sum\limits_{n=1}^{\infty}\dfrac{2^n}{e^{n-1}} = \sum\limits_{n=1}^{\infty}\dfrac{2^n}{e^{-1}e^n}$

$= e\sum\limits_{n=1}^{\infty}(\dfrac{2}{e})^n = e\cdot\dfrac{2/e}{1-(2/e)} = \dfrac{2}{(e-2)/e}$ $\sum\limits_{n=1}^{\infty}(\dfrac{2}{e})^n$ is an infinite geometric series with $a = \dfrac{2}{e}$ and $r = \dfrac{2}{e}$.

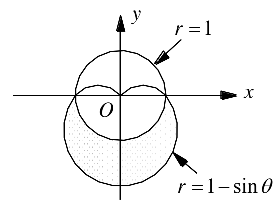
$= \dfrac{2e}{e-2}$

12. A $\int \dfrac{1}{x^2+2x+2}\,dx = \int \dfrac{1}{1+(x+1)^2}\,dx$

$= \arctan(x+1) + C$

13. E The points of intersection of the two curves are $(1,\pi)$ and $(1,2\pi)$. The shaded area can be found by subtracting the area inside the circle between $\theta=\pi$ and $\theta=2\pi$ from the area inside the cardoid between $\theta=\pi$ and $\theta=2\pi$.

Area $=\frac{1}{2}\int_{\pi}^{2\pi}\left[(1-\sin\theta)^2-1\right]d\theta$



14. C As $\lim_{t\to\infty}\frac{dP}{dt}=0$ for a logistic differential equation, this means that $(2-\frac{P}{60})=0$.
Therefore $\lim_{t\to\infty}P(t)=120$.

15. B $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{2(t+3)}{e^t}$

$\left.\frac{dy}{dx}\right|_{t=0}=\frac{2(0+3)}{e^0}=6$

16. D $x'(t)=e^t+1 \Rightarrow x(t)=\int(e^t+1)\,dt=e^t+t+C$

$x(0)=2 \Rightarrow e^0+0+C=2 \Rightarrow C=1$

$x(t)=e^t+t+1 \Rightarrow x(1)=e^1+1+1=e+2$

$y=\ln(\sqrt{x})$, $y(e+2)=\ln(\sqrt{e+2})=\frac{1}{2}\ln(e+2)$

17. C $x(t)=t^2+1 \Rightarrow x'(t)=2t \Rightarrow x'(2)=4$

$y(t)=te^{t/2} \Rightarrow y'(t)=\frac{1}{2}te^{t/2}+e^{t/2} \Rightarrow y'(2)=e+e=2e$

Speed $=|v|=\sqrt{[x'(t)]^2+[y'(t)]^2}$

Speed at time $t=2$ is

$\sqrt{[x'(2)]^2+[y'(2)]^2}=\sqrt{[4]^2+[2e]^2}=6.75$.

18. A $\int_1^\infty\frac{1}{\sqrt[p]{x}}\,dx$ converges if $\sum_{n=1}^{\infty}\frac{1}{\sqrt[p]{n}}$ converges.

$\sum_{n=1}^{\infty}\frac{1}{\sqrt[p]{n}}=\sum_{n=1}^{\infty}\frac{1}{n^{1/p}}$ convergent if $\frac{1}{p}>1 \Rightarrow p<1$.

19. D Let $u=\ln x$ and $dv=\frac{dx}{x^2}$. Then $du=\frac{1}{x}dx$ and $v=\int\frac{1}{x^2}\,dx=-\frac{1}{x}$.

$\int\frac{\ln x}{x^2}\,dx=(\ln x)(-\frac{1}{x})-\int(-\frac{1}{x})(\frac{1}{x}\,dx)=-\frac{\ln x}{x}+\int\frac{1}{x^2}\,dx$

$=-\frac{\ln x}{x}-\frac{1}{x}+C$

20. E I. Disregarding all but the highest powers of n in the numerator and the denominator, we can compare the series with $\sum_{n=1}^{\infty} \frac{1}{n}$. Let $a_n = \frac{n-2}{n(n+7)}$ and $b_n = \frac{1}{n}$.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n-2}{n(n+7)}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n-2}{n+7} = 1$$

Since $\sum b_n = \sum 1/n$ is a divergent harmonic series, the given series diverges by the Limit Comparison Test.

II. $\sum_{n=1}^{\infty} \frac{3^n+4^n}{5^n} = \sum_{n=1}^{\infty} (\frac{3^n}{5^n} + \frac{4^n}{5^n}) = \sum_{n=1}^{\infty} (\frac{3}{5})^n + \sum_{n=1}^{\infty} (\frac{4}{5})^n$

$\sum_{n=1}^{\infty} (\frac{3}{5})^n$ is a convergent geometric series with $r = \frac{3}{5} < 1$ and $\sum_{n=1}^{\infty} (\frac{4}{5})^n$ is also a convergent geometric series with $r = \frac{4}{5} < 1$. Therefore $\sum_{n=1}^{\infty} \frac{3^n+4^n}{5^n}$ is convergent.

III. $\sum_{n=1}^{\infty} ne^{-n} = \sum_{n=1}^{\infty} \frac{n}{e^n}$. Use the Ratio Test.

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)/e^{n+1}}{n/e^n}\right| = \frac{1}{e} \lim_{n\to\infty} \left|\frac{n+1}{n}\right| = \frac{1}{e} < 1,$$

so the series converges.

21. C $\frac{d}{dx}(\arcsin e^{x/2}) = \frac{1}{\sqrt{1-(e^{x/2})^2}} \cdot \frac{d}{dx}(e^{x/2}) = \frac{1}{\sqrt{1-e^x}}(\frac{1}{2}e^{x/2})$

$$= \frac{e^{x/2}}{2\sqrt{1-e^x}}$$

22. B Let $a_n = \frac{(4x-1)^n}{n4^n}$. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(4x-1)^{n+1}/(n+1)4^{n+1}}{(4x-1)^n/n4^n}\right| = \lim_{n\to\infty} \left|\frac{(4x-1)n}{4(n+1)}\right| = \left|\frac{4x-1}{4}\right|$.

The series converges when $\left|\frac{4x-1}{4}\right| < 1 \Rightarrow |4x-1| < 4 \Rightarrow -4 < 4x-1 < 4 \Rightarrow \frac{-3}{4} < x < \frac{5}{4}$.

We must now test for convergence at the end point of this interval.

When $x = -\frac{3}{4}$, the series becomes $\sum_{n=1}^{\infty} \frac{(4(-3/4)-1)^n}{n4^n} = \sum_{n=1}^{\infty} \frac{(-4)^n}{n4^n} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$, which is a convergent Alternating Series, decreasing in absolute value, and with limit 0.

When $x = \frac{5}{4}$, the series becomes $\sum_{n=1}^{\infty} \frac{(4(5/4)-1)^n}{n4^n} = \sum_{n=1}^{\infty} \frac{(4)^n}{n4^n} = \sum_{n=1}^{\infty} \frac{1}{n}$, which is a divergent p-series.

So the interval of convergence is $-\frac{3}{4} \le x < \frac{5}{4}$.

23. D The graph of f is continuous and passes through $(-1,7)$, $(5,7)$, and $(8,-2)$.

I. Not true. If the graph concaves down in the open interval $(-1,5)$, there cannot be a point such that $f(k)=3$.

II. True. f has at least one zero for $5<x<8$ by the Intermediate Value Theorem.

III. True. The Mean Value Theorem guarantees that there exists a number c on $(-1,5)$ such that $f'(c)=\dfrac{f(5)-f(-1)}{5-(-1)}=\dfrac{7-7}{6}=0$, so the graph of f has at least one horizontal tangent at $(-1,5)$.

24. D $\dfrac{dy}{dx}=y^2+1 \Rightarrow \dfrac{dy}{y^2+1}=dx \Rightarrow \displaystyle\int \dfrac{dy}{y^2+1}=\int dx \Rightarrow \arctan(y)=x+C$

$y(1)=0 \Rightarrow \arctan(0)=1+C \Rightarrow C=-1$

$\arctan(y)=x-1 \Rightarrow y=\tan(x-1)$

25. E $\displaystyle\int_1^2 f''(t)\,dt=f'(2)-f'(1)$ Fundamental Theorem of Calculus

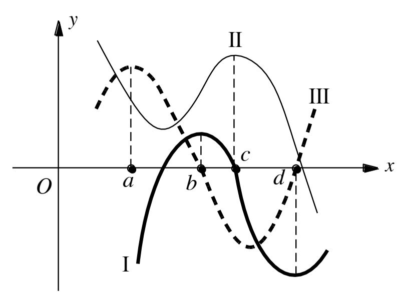
$=2-(-3)=5$ $f'(2)=2$ and $f'(1)=-3$

26. B Check each answer choice.

(A) If I is the graph of f, then $f'(b)=0$ $\Rightarrow$ III must be the graph of f'. If III is the graph of f', then II must have a 0 at $x=a$, but graph II doesn't have a 0 at $x=a$.

(B) If II is the graph of f, then $f'(c)=0 \Rightarrow$ I must be the graph of f'. If I is the graph of f', then III must have a 0 at $x=b$ and $x=d$.

Choice (B) is correct.



27. C $\displaystyle\int_0^\infty xe^{-x^2}\,dx=\lim_{b\to\infty}\int_0^b xe^{-x^2}\,dx=\lim_{b\to\infty}\left[-\frac{1}{2}e^{-x^2}\right]_0^b$

$=-\dfrac{1}{2}\displaystyle\lim_{b\to\infty}\left[e^{-b^2}-e^0\right]=\dfrac{1}{2}$

28. B A series expansion of $\cos x$ is $\cos x=1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}-\dfrac{x^6}{6!}+\cdots$.

$$\frac{1-\cos\sqrt{x}}{x}=\frac{1-(1-\frac{(\sqrt{x})^2}{2!}+\frac{(\sqrt{x})^4}{4!}-\frac{(\sqrt{x})^6}{6!}+\cdots)}{x}=\frac{\frac{x}{2!}-\frac{x^2}{4!}+\frac{x^3}{6!}-\cdots}{x}$$

$$=\frac{1}{2!}-\frac{x}{4!}+\frac{x^2}{6!}+\cdots+\frac{(-1)^{n-1}x^{n-1}}{(2n)!}+\cdots$$

SECTION I, Part B

76. D Average rate $= \dfrac{C(6)-C(0)}{6-0} = \dfrac{12e^{0.112\times 6} - 12e^{0}}{6} = 1.916$

77. A $f(x) = \sqrt{x\sin x}$, $f'(x) = \dfrac{1}{2}(x\sin x)^{-1/2}(x\cos x + \sin x) = \dfrac{x\cos x + \sin x}{2\sqrt{x\sin x}}$

$$f'(\frac{\pi}{2}) = \frac{(\pi/2)\cos(\pi/2) + \sin(\pi/2)}{2\sqrt{(\pi/2)\sin(\pi/2)}} = \frac{1}{2\sqrt{(\pi/2)}} = \frac{1}{\sqrt{2\pi}}$$

78. C $f(x) = x^2 + 3x - 1 \;\Rightarrow\; f'(x) = 2x+3$

$f(1) = 1+3-1 = 3$ and $f'(1) = 2+3 = 5$

An equation for the tangent line is $y-3 = 5(x-1)$ or $y = 5x-2$.

The difference between the function and the tangent line is

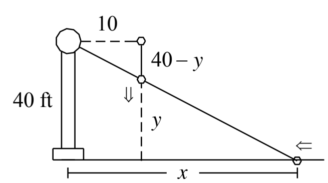
$(x^2+3x-1)-(5x-2) = x^2 - 2x + 1$.

Solve $x^2 - 2x + 1 < 0.3 \;\Rightarrow\; (x-1)^2 < 0.3 \;\Rightarrow\; -\sqrt{0.3} < x-1 < \sqrt{0.3}$

$\Rightarrow\; 1-\sqrt{0.3} < x < 1+\sqrt{0.3} \;\Rightarrow\; 0.452 < x < 1.547$

The largest value in the answer choice that satisfies the inequality is 1.5.

79. D $y(t) = 40 - 16\,t^2 \;\Rightarrow\; \dfrac{dy}{dt} = -32t$



$y(1) = 40-16 = 24$, $\left.\dfrac{dy}{dt}\right|_{t=1} = -32$

By similar triangles, $\dfrac{10}{40-y} = \dfrac{x-10}{y}$.

$10y = (40-y)(x-10) \;\Rightarrow\; 40x - xy = 400$

$40\dfrac{dx}{dt} - (x\dfrac{dy}{dt} + y\dfrac{dx}{dt}) = 0$. When $t=1$, $\dfrac{10}{40-24} = \dfrac{x-10}{24} \;\Rightarrow\; x = 25$

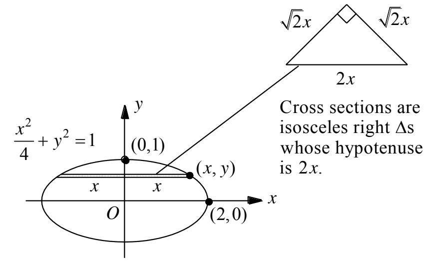
$$40\frac{dx}{dt} - \left[25(-32) + (24)\frac{dx}{dt}\right] = 0$$

$$\Rightarrow\; \frac{dx}{dt} = -50$$

80. A The area of a cross section is $A(x) = \dfrac{1}{2}(\sqrt{2}x)(\sqrt{2}x) = x^2$

$= 4(1-y^2)$.

The triangles lie on the plane from $y=-1$ to $y=1$.



$$V = \int_{-1}^{1} 4(1-y^2)\,dy = 8\int_{0}^{1}(1-y^2)\,dy = 8\left[y - \frac{y^3}{3}\right]_0^1$$

$$= \frac{16}{3}$$

81. D $g(4)-g(1)=\int_1^4 \frac{3x^2}{x^4+5}\,dx = 1.296$

$g(1)=g(4)-1.296=6-1.296=4.704$

82. C $r=1+3\sin\theta \Rightarrow \frac{dr}{d\theta}=3\cos\theta$

$\left.\frac{dr}{d\theta}\right|_{\theta=\frac{5\pi}{6}} = 3\cos(\frac{5\pi}{6}) = -\frac{3\sqrt{3}}{2} < 0$

Since r is positive and $\frac{dr}{d\theta}$ is negative when $\theta=\frac{5\pi}{6}$, the curve is getting closer to the origin.

83. E $\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots+\frac{(-1)^n x^{2n}}{(2n)!}+\cdots$

$x\cos x = x-\frac{x^3}{2!}+\frac{x^5}{4!}-\frac{x^7}{6!}+\cdots+\frac{(-1)^n x^{2n+1}}{(2n)!}+\cdots$

Use a graphing calculator to find the intersection of $\ln x$ and $x\cos x$.
The curves intersect at $x=1.347$.

84. D $s(t)=t^3-15t^2+14 \Rightarrow v(t)=3t^2-30t=3t(t-10) \Rightarrow a(t)=6t-30=6(t-5)$

	0 to 5	5 to 10	> 10
Signs of $v(t)$	−	−	+
Signs of $a(t)$	−	+	+

The particle's speed is increasing when $v(t)$ and $a(t)$ have the same sign.
That is, when $0<t<5$ and $t>10$.

85. B $y=x\ln x$, $\frac{dy}{dx}=x\cdot\frac{1}{x}+\ln x = 1+\ln x$

$L=\int_1^2 \sqrt{1+\left[\frac{dy}{dx}\right]^2}\,dx = \int_1^2 \sqrt{1+[1+\ln x]^2}\,dx = 1.713$

86. D $h(x)=\int_0^x 4(x-2)\cos(\frac{x}{2})\,dx \Rightarrow h'(x)=4(x-2)\cos(\frac{x}{2})$

I. Not true. On $(0,2)$ h' is negative, so h is decreasing.

II. True. $h'(3)=4(3-2)\cos(\frac{3}{2})=0.283>0$

III. True $h(3)=\int_0^3 4(x-2)\cos(\frac{x}{2})\,dx = -6.888<0$

87. A $v(t)=t^3-5t^2+2t+8$

The particle moves downward when v is negative. v is negative when $2<t<4$.

Displacement $=\int_a^b v(t)\,dt = \int_2^4 (t^3-5t^2+2t+8)\,dt$.

88. D Since the number of bacteria increases at a rate proportional to the number present, we can use the equation $y = y_0 e^{kt}$.

The colony starts with one bacterium and doubles every half-hour $\Rightarrow$ $2 = 1 \cdot e^{k(1/2)}$.

$\Rightarrow \ln 2 = \frac{1}{2}k \Rightarrow k = 1.38629$

At the end of 12 hours, $y = 1 \cdot e^{(1.38629)(12)} \approx 1.68 \times 10^7$

89. D Given $\frac{dy}{dx} = 1 - \frac{xy}{2}$, $x_0 = 0$, $y_0 = 1$, and $h = 0.5$.

$$f(0.5) \approx y_1 = y_0 + h\left[\frac{dy}{dx}\right]_{(x_0, y_0)} = 1 + (0.5)\left[1 - \frac{xy}{2}\right]_{(0,1)}$$

$$= 1 + (0.5)(1 - \frac{0}{2}) = 1.5 \qquad x_0 = 0,\ y_0 = 1$$

$$f(1) \approx y_2 = y_1 + h\left[\frac{dy}{dx}\right]_{(x_1, y_1)} = 1.5 + (0.5)\left[1 - \frac{xy}{2}\right]_{(0.5,1.5)}$$

$$= 1.5 + (0.5)(1 - \frac{(0.5)(1.5)}{2}) = 1.8125 \qquad x_1 = 0 + 0.5 = 0.5,\ y_1 = 1.5$$

90. B $x(t) = \arcsin t \Rightarrow \frac{dx}{dt} = \frac{1}{\sqrt{1-t^2}}$

$$y(t) = \ln\sqrt{1-t^2} \Rightarrow \frac{dy}{dt} = \frac{1}{\sqrt{1-t^2}} \cdot \frac{-2t}{2\sqrt{1-t^2}} = \frac{-t}{1-t^2}$$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_{-1/2}^{1/2} \sqrt{\left(\frac{1}{\sqrt{1-t^2}}\right)^2 + \left(\frac{-t}{1-t^2}\right)^2}\, dt = 1.099$$

91. C $r = 3 + \sin 5\theta$

$x = r\cos\theta = (3 + \sin 5\theta)\cos\theta = 2$

Use a graphing calculator. In function mode, enter $\boxed{y_1 = (3 + \sin(5x))\cos x}$, $\boxed{y_2 = 2}$ and find the point of intersection.

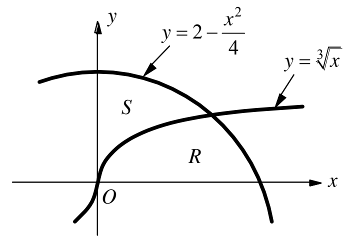
$\theta = 0.705$

92. A $P(x) = x - 2x^2 + 2x^3 - \frac{4}{3}x^4$

$$\frac{f^{(4)}(0)}{4!} = -\frac{4}{3} \Rightarrow f^{(4)}(0) = -\frac{4}{3} \cdot 4! = -32$$

SECTION II, Part A

1. (a) Find the point of intersection using a graphing calculator. Point of intersection is $(1.776, 1.211)$. The x-intercept of $y = 2 - \dfrac{x^2}{4}$ is $(2\sqrt{2}, 0)$.



Area of $R = \int_0^{1.776} \sqrt[3]{x}\, dx + \int_{1.776}^{2\sqrt{2}} (2 - \frac{x^2}{4})\, dx$

$= 1.613 + 0.686 = 2.299$

(b) Area of $S = \int_0^{1.776} \left[(2 - \frac{x^2}{4}) - \sqrt[3]{x} \right] dx = 1.472$

(c) $V = \int_0^{1.776} \left[\sqrt[3]{x}\right]^2 dx + \int_{1.776}^{2\sqrt{2}} \left[2 - \frac{x^2}{4} \right]^2 dx = 2.139$

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2. (a) $v(t) = t\ln(t^2) \Rightarrow a(t) = t \cdot \frac{2t}{t^2} + \ln t^2 = 2 + \ln t^2$

$a(0.5) = 2 + \ln(0.5)^2 = 2 - 1.386 = 0.614$

(b) $v(0.5) = (0.5)\ln(0.5)^2 = -0.693 < 0$ and $a(0.5) = 0.614 > 0$.
The speed of the particle is decreasing because at time $t = 0.5$, $v(t)$ and $a(t)$ have opposite signs.

(c) $v(t) = t\ln(t^2)$

Signs of $v(t)$	0 to 1	1 and beyond
	$-$	$+$

The particle is moving to the left for $0 \le t \le 1$, thus at time $t = 1$, the particle is farthest to the left.

$x(1) - x(0) = \int_0^1 v(t)\, dt = \int_0^1 t\ln(t^2)\, dt = -0.5$

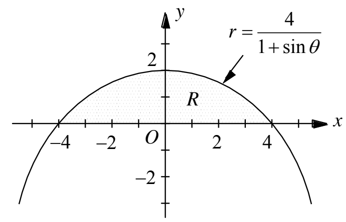
$x(1) = x(0) - 0.5 = -1 - 0.5 = -1.5$

The distance between the particle and the origin is 1.5, when the particle is farthest to the left.

(d) $x(0.5) - x(0) = \int_0^{0.5} v(t)\, dt = \int_0^{0.5} t\ln(t^2)\, dt = -0.298$

$x(0.5) = x(0) - 0.298 = -1.298$

At time $t = 0.5$, the particle is to the left of the origin and moving to the left since $v(0.5) < 0$. Therefore the particle is moving away from the origin at time $t = 0.5$.



3. (a) $r=\dfrac{4}{1+\sin\theta} \Rightarrow \dfrac{dr}{d\theta}=\dfrac{-4\cos\theta}{(1+\sin\theta)^2}$

$$\left.\frac{dr}{d\theta}\right|_{\theta=\frac{\pi}{6}}=\frac{-4\cos(\pi/6)}{(1+\sin(\pi/6))^2}=-\frac{2\sqrt{3}}{(1+1/2)^2}=-\frac{8\sqrt{3}}{9}$$

The curve is getting closer to the origin when $\theta=\dfrac{\pi}{6}$ because r is positive and $\dfrac{dr}{d\theta}$ is negative at $\theta=\dfrac{\pi}{6}$.

(b) $A=\dfrac{1}{2}\displaystyle\int_0^{\pi} r^2\,d\theta=\dfrac{1}{2}\int_0^{\pi}\left(\dfrac{4}{1+\sin\theta}\right)^2 d\theta=\dfrac{32}{3}$

(c) $r=\dfrac{4}{1+\sin\theta} \Rightarrow r+r\sin\theta=4 \Rightarrow r+y=4$ — Make substitution. $y=r\sin\theta$

$\Rightarrow r=4-y \Rightarrow r^2=(4-y)^2 \Rightarrow x^2+y^2=(4-y)^2$ — Make substitution. $x^2+y^2=r^2$

$\Rightarrow x^2+y^2=16-8y+y^2 \Rightarrow x^2=16-8y$

$\Rightarrow y=-\dfrac{1}{8}x^2+2$

(d) $A=\displaystyle\int_{-4}^{4}\left(-\frac{1}{8}x^2+2\right)dx=2\int_0^4\left(-\frac{1}{8}x^2+2\right)dx=\int_0^4\left(-\frac{1}{4}x^2+4\right)dx$

$=\left[-\dfrac{1}{12}x^3+4x\right]_0^4=\dfrac{32}{3}$

SECTION II, Part B

t (hours)	0	3	6	9	12	15	18	21	24
$P(t)$ (gallons / hour)	680	620	760	1040	1200	1120	960	920	680

4. (a) $P'(7.5)\approx\dfrac{P(9)-P(6)}{9-6}=\dfrac{1040-760}{3}=93.3$ gallons / hr^2

(b) $P''(7.5)=0$ since $P'(t)$ has a maximum at $t=7.5$.

(c) Average value $=\dfrac{1}{24-12}\displaystyle\int_{12}^{24}P(t)\,dt$

$\approx\dfrac{1}{12}\left[3(1200)+3(1120)+3(960)+3(920)\right]$

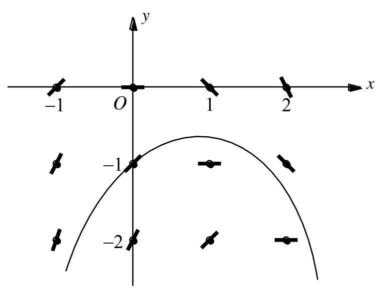
$=1050$ gallons / hour

(d) Average rate of fuel consumption on $[15,18]$ is $\dfrac{960-1120}{18-15}=-\dfrac{160}{3}$.

Average rate of fuel consumption on $[18,21]$ is $\dfrac{920-960}{21-18}=-\dfrac{40}{3}$.

No. By the Mean Value Theorem, $P'(c_1)=-\dfrac{160}{3}$ for some c_1 in the interval $(15,18)$ and $P'(c_2)=-\dfrac{40}{3}$ for some c_2 in the interval $(18,21)$. It follows that P' must increase somewhere in the interval (c_1,c_2). Therefore P'' is not negative for every x in $(12,24)$.

5 (a)



x	-1	-1	-1	0	0	0	1	1	1	2	2	2
y	0	-1	-2	0	-1	-2	0	-1	-2	0	-1	-2
$y'=-x-y$	1	2	3	0	1	2	-1	0	1	-2	-1	0

(b) Given $\dfrac{dy}{dx}=-x-y$, $f(0)=-1$, and $h=-0.2$.

$$f(-0.2)\approx y_1=y_0+h\left[\frac{dy}{dx}\right]_{(x_0,y_0)}=-1+(-0.2)\left[-x-y\right]_{(0,-1)}$$
$$=-1+(-0.2)(0+1)=-1.2$$

$$f(-0.4)\approx y_2=y_1+h\left[\frac{dy}{dx}\right]_{(x_1,y_1)}=-1.2+(-0.2)\left[-x-y\right]_{(-0.2,-1.2)}$$
$$=-1.2+(-0.2)(0.2+1.2)=-1.48$$

(c) At local maximum $\dfrac{dy}{dx}=0$.

$$\frac{dy}{dx}=-x-y=0 \Rightarrow y=-x \Rightarrow y=-\ln 2$$

The y-coordinate is $-\ln 2$.

(d) $\dfrac{d^2y}{dx^2}=\dfrac{d}{dx}\left[-x-y\right]=-1-\dfrac{dy}{dx}=-1-(-x-y)=-1+x+y$

$\dfrac{d^2y}{dx^2}$ is negative because $x(=-0.4)$ and $y(\approx -1.48)$ are both negative. Thus the graph concaves down and the approximation is greater than $f(-0.4)$.

6. (a) $f(x)=1-\frac{x^2}{3}+\frac{x^4}{5}-\frac{x^6}{7}+\cdots+\frac{(-1)^{n-1}x^{2n-2}}{2n-1}+\cdots$

$$f'(x)=-\frac{2x}{3}+\frac{4x^3}{5}-\frac{6x^5}{7}+\cdots+\frac{(-1)^n(2n)x^{2n-1}}{2n+1}+\cdots$$

(b) $f'(\frac{1}{\sqrt{3}})=-\frac{2}{3}\cdot\frac{1}{\sqrt{3}}+\frac{4}{5}\cdot\frac{1}{3\sqrt{3}}-\frac{6}{7}\cdot\frac{1}{3^2\sqrt{3}}+\cdots+\frac{(-1)^n(2n)}{2n+1}\cdot\frac{1}{3^{n-1}\sqrt{3}}+\cdots$

$$f(x)=\frac{\tan^{-1}x}{x} \Rightarrow f'(x)=\frac{x\cdot\frac{1}{1+x^2}-\tan^{-1}x}{x^2}$$

$$f'(\frac{1}{\sqrt{3}})=\frac{\frac{1}{\sqrt{3}}\cdot(\frac{1}{1+(1/\sqrt{3})^2})-\tan^{-1}\frac{1}{\sqrt{3}}}{(1/\sqrt{3})^2}=\frac{\frac{1}{\sqrt{3}}(\frac{3}{4})-(\pi/6)}{1/3}=\frac{3\sqrt{3}}{4}-\frac{\pi}{2}$$

Therefore $-\frac{2}{3}\cdot\frac{1}{\sqrt{3}}+\frac{4}{5}\cdot\frac{1}{3\sqrt{3}}-\frac{6}{7}\cdot\frac{1}{3^2\sqrt{3}}+\cdots+\frac{(-1)^n(2n)}{2n-1}\cdot\frac{1}{3^{n-1}\sqrt{3}}+\cdots=\frac{3\sqrt{3}}{4}-\frac{\pi}{2}$.

(c) $g(x)=\int_0^x f(t)\,dt=\int_0^x (1-\frac{t^2}{3}+\frac{t^4}{5}-\frac{t^6}{7}+\cdots+\frac{(-1)^{n-1}t^{2n-2}}{2n-1}+\cdots)\,dx$

$$=x-\frac{x^3}{3^2}+\frac{x^5}{5^2}-\frac{x^7}{7^2}+\cdots+\frac{(-1)^{n-1}x^{2n-1}}{(2n-1)^2}+\cdots$$

(d) $g(1)=1-\frac{1}{3^2}+\frac{1}{5^2}-\frac{1}{7^2}+\cdots+\frac{(-1)^{n-1}}{(2n-1)^2}+\cdots$

$g(1)$ decreases in absolute value, and has limit 0. By the Alternating Series Estimation theorem

$$\left|g(1)-(1-\frac{1}{3^2}+\frac{1}{5^2})\right|\le\frac{1}{7^2}<\frac{1}{40}$$

Answer Key

SECTION I, Part A

1. A	2. C	3. B	4. C	5. C	6. B	7. D	8. D	9. A	10. D
11. A	12. B	13. E	14. E	15. B	16. C	17. D	18. E	19. C	20. D
21. E	22. C	23. D	24. A	25. D	26. D	27. B	28. C		

SECTION I, Part B

76. C	77. C	78. D	79. E	80. E	81. A	82. C	83. D	84. C	85. A
86. B	87. D	88. C	89. B	90. D	91. C	92. A			

SECTION I, Part A

1. A $\lim_{x\to 0}\dfrac{1-\cos x}{1+x-e^x}$ Indeterminate form $\dfrac{0}{0}$.

$=\lim_{x\to 0}\dfrac{\sin x}{1-e^x}$ Apply L'hopital's rule.

$=\lim_{x\to 0}\dfrac{\cos x}{-e^x}=-1$ Apply L'hopital's rule and substitute $x=0$.

2. C $g(x)=f\left(f(x)\right) \Rightarrow g'(x)=f'\left(f(x)\right)\cdot f'(x)=f'\left(e^{-x}\right)\cdot f'(x)$

$g'(0)=f'\left(e^{-0}\right)\cdot f'(0)=f'(1)\cdot f'(0)$

$f(x)=e^{-x} \Rightarrow f'(x)=-e^{-x}$

$f'(1)=-e^{-1}=-\dfrac{1}{e}$ and $f'(0)=-e^{-0}=-1$

Therefore $g'(0)=f'(1)\cdot f'(0)=(-\dfrac{1}{e})(-1)=\dfrac{1}{e}$.

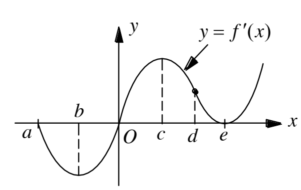
3. B $3\sin x\cos y=1 \Rightarrow \sin x\cos y=\dfrac{1}{3} \Rightarrow \sin x\cdot\dfrac{d}{dx}(\cos y)+\cos y\cdot\dfrac{d}{dx}(\sin x)=0$

$\Rightarrow \sin x(-\sin y\cdot\dfrac{dy}{dx})+\cos y(\cos x)=0 \Rightarrow \dfrac{dy}{dx}=\dfrac{\cos x\cos y}{\sin x\sin y}$

$\Rightarrow \dfrac{dy}{dx}=\cot x\cot y$

4. C f' is increasing for $b<x<c$ and $x>e$.

Therefore f concaves up for $b<x<c$ and $x>e$.



5. C $h(x) = \arctan x + \arctan(\frac{1}{x})$

$$\Rightarrow \; h'(x) = \frac{1}{1+x^2} + \frac{1}{1+(1/x)^2} \cdot \frac{d}{dx}(\frac{1}{x}) = \frac{1}{1+x^2} + \frac{1}{1+1/x^2} \cdot (-\frac{1}{x^2}) = 0$$

6. B $f(x) = \int_{-1}^{2x^4-x} e^t \; dt \;\Rightarrow\; f'(x) = e^{2x^4-x} \cdot \frac{d}{dx}(2x^4 - x) = e^{2x^4-x}(8x^3 - 1)$

$$f'(x) = 0 \;\Rightarrow\; 8x^3 - 1 = 0 \;\Rightarrow\; x = \frac{1}{2}$$

7. D $\lim\limits_{h \to 0} \frac{\tan(\frac{\pi}{4}+h) - 1}{h} = \lim\limits_{h \to 0} \frac{\tan(\frac{\pi}{4}+h) - \tan(\frac{\pi}{4})}{h} = f'(\frac{\pi}{4})$, where $f(x) = \tan x$.

$$f'(x) = \sec^2 x \;\Rightarrow\; f'(\frac{\pi}{4}) = \sec^2 \frac{\pi}{4} = (\sqrt{2})^2 = 2$$

8. D $f(x) = \begin{cases} x^3 & \text{for } x < 0, \\ \sin 2x & \text{for } x \geq 0, \end{cases}$

$$\int_{-1}^{\pi/2} f(x)\, dx = \int_{-1}^{0} x^3 \; dx + \int_{0}^{\pi/2} \sin(2x)\, dx = \left[\frac{x^4}{4}\right]_{-1}^{0} + \left[-\frac{1}{2}\cos(2x)\right]_{0}^{\pi/2}$$

$$= \left[0 - \frac{1}{4}\right] - \frac{1}{2}\left[\cos\pi - \cos 0\right] = -\frac{1}{4} - \frac{1}{2}(-2) = \frac{3}{4}$$

9. A $y = 1 + 4x^{3/2} \;\Rightarrow\; y' = 6x^{1/2}$

$$L = \int_a^b \sqrt{1+[y']^2}\; dx = \int_0^3 \sqrt{1+\left[6x^{1/2}\right]^2}\; dx = \int_0^3 \sqrt{1+36x}\; dx$$

10. D Average value $= \frac{1}{b-a}\int_a^b f(x)\, dx = \frac{1}{k-0}\int_0^k \sqrt{x}\; dx = \frac{1}{k}\left[\frac{2}{3}x^{3/2}\right]_0^k = \frac{1}{k}\left[\frac{2}{3}k^{3/2}\right] = \frac{2}{3}k^{1/2}$

$$\frac{2}{3}k^{1/2} = 2 \;\Rightarrow\; k = 9$$

11. A $x = 3\sec\theta \;\Rightarrow\; dx = 3\sec\theta\tan\theta\; d\theta$

$$\int \frac{\sqrt{x^2-9}}{x^3}\; dx = \int \frac{\sqrt{(3\sec\theta)^2-9}}{(3\sec\theta)^3}\;(3\sec\theta\tan\theta)\; d\theta = \int \frac{3\sqrt{\sec^2\theta - 1}}{27\sec^3\theta}\;(3\sec\theta\tan\theta)\; d\theta$$

$$= \frac{1}{3}\int \frac{\sqrt{\tan^2\theta}\;(\sec\theta\tan\theta)}{\sec^3\theta}\; d\theta = \frac{1}{3}\int \frac{\tan^2\theta}{\sec^2\theta}\; d\theta = \frac{1}{3}\int \sin^2\theta\; d\theta$$

12. B In logistic equation $\frac{dP}{dt} = kP\left(1 - \frac{P}{A}\right)$, the population is growing the fastest when $P = \frac{A}{2}$.

$$\frac{dP}{dt} = 2P\left(1 - \frac{P}{90}\right) \;\Rightarrow\; A = 90 \;\Rightarrow\; \frac{A}{2} = 45$$

13. E $\int_0^{\infty} \frac{x}{x^2+4}\,dx = \lim_{b\to\infty}\int_0^b \frac{x}{x^2+4}\,dx = \lim_{b\to\infty}\left[\frac{1}{2}\ln(x^2+4)\right]_0^b = \frac{1}{2}\lim_{b\to\infty}\left[\ln(x^2+4)\right]_0^b$

$= \frac{1}{2}\lim_{b\to\infty}\left[\ln(b^2+4)-\ln 4\right] = \infty$

14. E $\frac{dy}{dx} = \frac{e^x}{3y^2} \Rightarrow 3y^2dy = e^x dx \Rightarrow \int 3y^2 dy = \int e^x dx \Rightarrow y^3 = e^x + C$

$y(0) = 2 \Rightarrow 2^3 = e^0 + C \Rightarrow C = 7$

$\Rightarrow y^3 = e^x + 7 \Rightarrow y = \sqrt[3]{e^x+7}$

15. B If f is continuous on $(-\infty,\infty)$, then $\lim_{x\to 2^-} f(x) = f(2) \Rightarrow \lim_{x\to 2^-}(x^2-a^2) = 2a+5$.

$4-a^2 = 2a+5 \Rightarrow a^2+2a+1 = 0 \Rightarrow (a+1)^2 = 0$

$\Rightarrow a = -1$

16. C Let $u = \ln(\sin x)$ and $dv = \cos x\,dx$, then $du = \frac{1}{\sin x}\cdot\cos x\,dx = \frac{\cos x}{\sin x}\,dx$ and $v = \sin x$.

$\int u\,dv = uv - \int v\,du = \ln(\sin x)(\sin x) - \int \sin x\frac{\cos x}{\sin x}\,dx = \sin x\ \ln(\sin x) - \int \cos x\,dx$

$= \sin x\ \ln(\sin x) - \sin x + C$

17. D $\lim_{h\to 0}\frac{\int_1^{1+h}(\tan^{-1}x)\,dx}{h} = \lim_{h\to 0}\frac{F(1+h)-F(1)}{h} = F'(1)$, where $F'(x) = \tan^{-1}x$.

$F'(1) = \tan^{-1}1 = \frac{\pi}{4}$

18. E If $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$ the curve has a vertical tangent.

$\frac{dx}{dt} = 3t^2 - 6t = 0 \Rightarrow 3t(t-2) = 0 \Rightarrow t = 0$ or $t = 2$

When $t = 0$ or $t = 2$ $\frac{dy}{dt} = 4t^3 - 7 \neq 0$, so the curve has a vertical tangent when $t = 0$ or $t = 2$.

19. C $x = \frac{t}{1+t} \Rightarrow \frac{dx}{dt} = \frac{(1+t)-t}{(1+t)^2} = \frac{1}{(1+t)^2}$

$y = \ln(1+t) \Rightarrow \frac{dy}{dt} = \frac{1}{1+t}$

$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_0^1 \sqrt{\frac{1}{(1+t)^4} + \frac{1}{(1+t)^2}}\,dt$

20. D I. $1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\cdots=\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$.

This is a p-series with $p>1$, so the series converges.

II. $\frac{2}{5}-\frac{3}{6}+\frac{4}{7}-\frac{5}{8}+\cdots=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n+1}{n+4}$.

Since $\lim_{x\to\infty} a_n \neq 0$, the series diverges by the nth Term Test for Divergence.

III. $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)!}{1\cdot 3\cdot 5\cdot\ \cdot\ \cdot(2n+1)(2n+3)}\cdot\frac{1\cdot 3\cdot 5\cdot\ \cdot\ \cdot(2n+1)}{n!}\right| = \lim_{x\to\infty}\left|\frac{n+1}{2n+3}\right| = \frac{1}{2}<1$.

By the Ratio Test, the series converges.

21. E $v(t)=e^{\sin t}+\sin(e^t) \Rightarrow a(t)=v'(t)=e^{\sin t}\cdot\cos t+\cos(e^t)\cdot e^t$

I. True. Since $v(1)=e^{\sin 1}+\sin(e^1)>0$, the particle is moving to the right at time $t=1$.

II. True. Since $a(1)=e^{\sin 1}\cdot\cos(1)+\cos(e^1)\cdot e^1<0$.

III. True At time $t=1$, $v(1)$ and $a(1)$ have opposite signs, therefore the particle's speed is decreasing.

22. C The desired area can be found by subtracting the area inside circle $r=\sin\theta$, between $\theta=0$ and $\theta=\pi$, from the area inside circle $r=2\sin\theta$, between $\theta=0$ and $\theta=\pi$.

$$A=\frac{1}{2}\int_0^{\pi}\left[(2\sin\theta)^2-(\sin\theta)^2\right]d\theta=\frac{1}{2}\int_0^{\pi}\left[3\sin^2\theta\right]d\theta$$

Since the region is symmetric about $\theta=\frac{\pi}{2}$, we can write

$$A=\frac{1}{2}\cdot 2\int_0^{\pi/2}\left[3\sin^2\theta\right]d\theta=3\int_0^{\pi/2}\sin^2\theta\,d\theta.$$

23. D $e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$

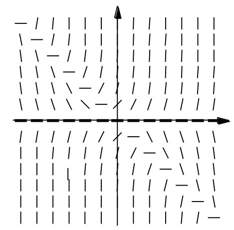
$$e^{-x}=1+(-x)+\frac{(-x)^2}{2!}+\frac{(-x)^3}{3!}+\cdots=1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots$$

The fourth-degree Taylor polynomial about $x=0$ of xe^{-x} is

$$xe^{-x}=x(1-x+\frac{x^2}{2!}-\frac{x^3}{3!})=x-x^2+\frac{x^3}{2}-\frac{x^4}{6}.$$

24. A If $\int_1^{\infty}\frac{\ln x}{x^2}\,dx=1$, then $\sum_{n=1}^{\infty}\frac{\ln n}{n^2}$ is convergent by the Integral Test. But the fact that the integral converges to 1 does not imply that the infinite series converges to 1.

25. D The slopes are zero at $(-2,2)$, $(-1,1)$, and $(1,-1)$, where $y=-x$. Eliminate choices (C) and (E).
Also the slopes are zero when $y=0$.
Choice (D) is correct.



26. D $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+1)^2 3^{n+1}}\cdot\frac{n^2 3^n}{x^n}\right| = \lim_{n\to\infty}\left|\frac{n^2}{(n+1)^2}\cdot\frac{x}{3}\right| = \left|\frac{x}{3}\right|$.

The series converges if $\left|\frac{x}{3}\right| < 1 \Rightarrow |x| < 3 \Rightarrow -3 < x < 3$.

We must now test for convergence at the end point of the interval.

When $x=-3$, the series becomes $\sum_{n=1}^{\infty}(-1)^n\frac{(-3)^n}{n^2 3^n} = \sum_{n=1}^{\infty}\frac{3^n}{n^2 3^n} = \sum_{n=1}^{\infty}\frac{1}{n^2}$, which is a convergent p-series with $p=2>1$.

When $x=3$, the series becomes $\sum_{n=1}^{\infty}(-1)^n\frac{3^n}{n^2 3^n} = \sum_{n=1}^{\infty}(-1)^n\frac{1}{n^2}$, which converges by the Alternating Series Test.

So the interval of convergence is $-3 \le x \le 3$.

27. B $g(x) = f^{-1}(x)$. $g(4)=1 \Rightarrow f^{-1}(4)=1$

$f(x) = x^4 - 2x + 5 \Rightarrow f'(x) = 4x^3 - 2 \Rightarrow f'(1) = 4-2 = 2$

$g'(4) = (f^{-1})'(4) = \frac{1}{f'(f^{-1}(4))} = \frac{1}{f'(1)} = \frac{1}{2}$

28. C We need to remember some frequently used series expansions.

(A) $x\sin x = x(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots) = x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \cdots$

(B) $x^2\cos x = x^2(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots) = x^2 - \frac{x^4}{2!} + \frac{x^6}{4!} - \cdots$

(C) $$\frac{\sin(x^2)}{x^2} = \frac{x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!} - \frac{(x^2)^7}{7!} + \cdots}{x^2} = \frac{x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \cdots}{x^2}$$

$$= 1 - \frac{x^4}{3!} + \frac{x^8}{5!} - \frac{x^{12}}{7!} + \cdots + \frac{(-1)^{n-1}x^{4(n-1)}}{(2n-1)!} + \cdots$$

Choice (C) is correct.

SECTION I, Part B

76. C $\int_1^4 \ln(x^2)\,dx \approx 1[f(1.5)+f(2.5)+f(3.5)] = \ln(1.5^2)+\ln(2.5^2)+\ln(3.5^2) = 5.149$

77. C Since the rate at which the amount of substance changes with time is proportional to the amount present, we can use the equation $y = y_0 e^{kt}$.

$70 = 100e^{k\cdot 10}$ 100 liters of chemical reduces to 70 liters during the first 10 hours.

$e^{10k} = \dfrac{7}{10} \Rightarrow \ln e^{10k} = \ln\dfrac{7}{10} \Rightarrow k = -0.035667$

$y = 100e^{-0.035667t}$

After 22 hours of the chemical reaction, $y = 100e^{-0.035667\times 22} \approx 45.6$.

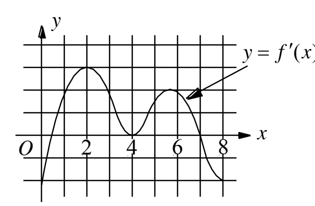
78. D $x_1 = -\dfrac{2}{3}\cos(\dfrac{3}{2}t) \Rightarrow v_1 = -\dfrac{2}{3}(-\dfrac{3}{2}\sin(\dfrac{3}{2}t)) = \sin(\dfrac{3}{2}t)$

$x_2 = \ln(3t) \Rightarrow v_2 = \dfrac{1}{3t}\cdot 3 = \dfrac{1}{t}$

Use a graphing calculator to find the intersection of $y_1 = \sin(\dfrac{3}{2}x)$ and $y_2 = \dfrac{1}{t}$, for $0 \le t \le 12$.

The two velocity functions intersect at 6 points, therefore the two particles have the same velocity 6 times for $0 \le t \le 12$.

79. E (A) f is increasing for $2 \le x \le 4$, because f' is positive for $2 \le x \le 4$.



(B) f is decreasing for $x \ge 7$, because f' is negative for $x \ge 7$.

(C) f concaves up for $0 \le x \le 2$, because f' is increasing for $0 \le x \le 2$.

(D) f concaves down for $2 \le x \le 4$, because f' is decreasing for $2 \le x \le 4$.

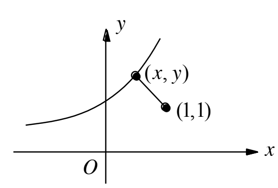
(E) f has a relative maximum at $x = 7$, because f' changes from positive to negative at $x = 7$.
f has a relative minimum for $0 \le x \le 1$, because f' changes from negative to positive for $0 \le x \le 1$.
f' does not change signs at $x = 4$, so f does not have a relative extremum at $x = 4$.
Therefore f has only one relative maximum for $0 \le x \le 8$.

80. E Let $u = 3 - x$, then $du = -dx$. $u = 3$ when $x = 0$ and $u = 0$ when $x = 3$.

$\int_0^3 f(3-x)\,dx = \int_3^0 f(u)\,(-du) = \int_0^3 f(u)\,du$

Since integrals do not depend on the variables that are used $\int_0^3 f(u)\,du = \int_0^3 f(x)\,dx = 5$.

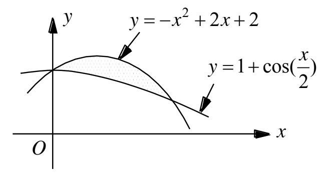
81. A $s = \sqrt{(x-1)^2 + (y-1)^2} = \sqrt{(x-1)^2 + (e^x - 1)^2}$



We need to find only the critical number of $s^2 = (x-1)^2 + (e^x - 1)^2$.

$\dfrac{d(s^2)}{dx} = 2(x-1) + 2(e^x - 1)e^x = 0 \Rightarrow x = 0.365$

82. C $y=-x^2+2x+2$, $y=1+\cos(\frac{x}{2})$



Find the points of intersection using a graphing calculator.
The points of intersection are $(0,2)$ and $(2.253, 1.429)$.

$$V=\pi\int_0^{2.253}\left[(-x^2+2x+2)^2-(1+\cos\frac{x}{2})^2\right]dx=24.445$$

83. D h is the rate of change of the altitude. Therefore when h is negative, the altitude of the balloon is decreasing.
$h(t)=4\sin(e^{x/3})+1<0$ for $3.666\le x\le 5.390$.
Change in altitude $=\int_{3.666}^{5.390}h(t)\,dt$. Distance traveled $=\int_a^b v(t)\,dt$.

84. C Let $r(t)=-80e^{-0.6t}$.
$r(3)-r(0)=\int_0^3(-80e^{-0.6t})\,dt \Rightarrow r(3)-220=-111.293$
$r(3)=220-111=109$
Note that the room temperature of 72°F is not used in answering the question because the cooling rate $-80e^{-0.6t}$ is decided by the room temperature.

85. A $v(6)-v(0)=\int_0^6 a(t)\,dt$
$\approx 2[f(1)+f(3)+f(5)]$
$=2[4+3+8]=30$ ft / sec
$v(6)=v(0)+30=-7+30=23$ ft / sec

t(sec)	0	1	2	3	4	5	6
$a(t)$ (ft /sec^2)	7	4	5	3	6	8	6

86. B $f(x)=e^x-x \Rightarrow f'(x)=e^x-1$
Average value $=\frac{1}{3-(-2)}\int_{-2}^3(e^x-x)\,dx=\frac{1}{5}(17.45)=3.49$
$f'(k)=e^k-1=3.49 \Rightarrow k=1.502$

87. D I. Let $a_n=\frac{1+n}{\sqrt[3]{1+n^6}}$ and $b_n=\frac{n}{\sqrt[3]{n^6}}=\frac{1}{n}$

$$\lim_{n\to\infty}\left|\frac{a_n}{b_n}\right|=\lim_{n\to\infty}\left|\frac{1+n}{\sqrt[3]{1+n^6}}\cdot\frac{n}{1}\right|=1$$

Since $\sum b_n=\sum\frac{1}{n}$ is a divergent harmonic series, the given series diverges by the Limit Comparison Test.

II. $\lim_{n\to\infty}3^{1/n}=3^0=1\ne 0$. The series diverges by the nth Term Test.

III. $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\left|\frac{2^{n+1}(n+1)}{3^n}\cdot\frac{3^{n-1}}{2^n n}\right|\lim_{n\to\infty}\left|\frac{2(n+1)}{3n}\right|=\frac{2}{3}<1$

So the series is absolutely convergent by the Ratio Test.

88. C $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

$\cos 1 \approx 1 - \frac{1}{2} + \frac{1}{24} - \frac{1}{720}$

89. B Given $x_0 = 1$, $y_0 = -1$, and $h = 0.3$.

x	1	1.1	1.2	1.3	1.4	1.5	1.6
$f'(x)$	−1.8	−1.6	−1.5	−1.2	0.8	1.2	1.5

$f(1.3) \approx y_1 = y_0 + h\left[\frac{dy}{dx}\right]_{x_0=1}$

$= -1 + 0.3(-1.8) = -1.54$

$f(1.6) \approx y_2 = y_1 + h\left[\frac{dy}{dx}\right]_{x_1=1.3}$

$= -1.54 + 0.3(-1.2) = -1.9$

90. D $f(x) = \sum_{n=1}^{\infty} (\sin^2 x)^n$

$f(2) = \sum_{n=1}^{\infty} (\sin^2 2)^n = (\sin^2 2)^1 + (\sin^2 2)^2 + (\sin^2 2)^3 + \cdots$ is an infinite geometric series with $a = \sin^2 2$ and $r = \sin^2 2$.

$f(2) = \frac{a}{1-r} = \frac{\sin^2 2}{1-\sin^2 2} = 4.774$

91. C $x(t) = t^3 - 3t^2 \Rightarrow x'(t) = 3t^2 - 6t = 3t(t-2)$

$y(t) = t - \ln t^2 \Rightarrow y'(t) = 1 - \frac{1}{t^2} \cdot 2t = 1 - \frac{2}{t}$

The particle is at rest when both $x'(t)$ and $y'(t)$ are equal to zero.
That is when $t = 2$.

92. A $f(x) = 4\sin(\frac{x}{2}) \Rightarrow f(ax) = 4\sin(\frac{ax}{2})$

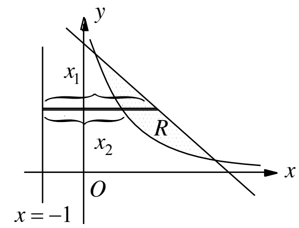
$f'(ax) = 4\cos(\frac{ax}{2}) \cdot \frac{a}{2} = 2a\cos(\frac{ax}{2}) \Rightarrow f''(ax) = 2a\left[-\sin(\frac{ax}{2}) \cdot \frac{a}{2}\right] = -a^2 \sin(\frac{ax}{2})$

$g(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots = \sin x$

$g(x) = k f''(ax) \Rightarrow \sin x = -ka^2 \sin(\frac{ax}{2}) \Rightarrow \frac{a}{2} = 1$ and $-ka^2 = 1$

$\Rightarrow a = 2 \Rightarrow -4k = 1 \Rightarrow k = -\frac{1}{4}$

SECTION II, Part A



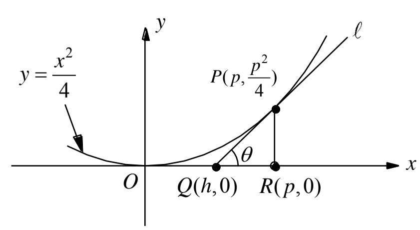
1. (a) Find the points of intersection using a graphing calculator.
The points of intersection are $(0.158, 3.841)$ and $(3.146, 0.853)$.

Area of $R = \int_{0.158}^{3.146} \left[(4-x)-(2-\ln x)\right] dx = 1.949$

(b) $V = \pi\int_{0.158}^{3.146} \left[(4-x+3)^2-(2-\ln x+3)^2\right] dx = 63.773$

(c) $y = 4-x \Rightarrow x = 4-y$, and $y = 2-\ln x \Rightarrow x = e^{2-y}$.

$V = \pi\int_{0.853}^{3.841} \left[{x_1}^2-{x_2}^2\right] dy = \pi\int_{0.853}^{3.841} \left[(4-y+1)^2-(e^{2-y}+1)^2\right] dy$



2. (a) $y = \dfrac{x^2}{4} \Rightarrow \dfrac{dy}{dx} = \dfrac{x}{2}$

Line ℓ passes through $(p, \dfrac{p^2}{4})$ and $(h,0)$ and has slope $\dfrac{p}{2}$.

Therefore, $\dfrac{\frac{p^2}{4}-0}{p-h} = \dfrac{p}{2} \Rightarrow \dfrac{p^2}{2} = p^2-ph$.

$h = \dfrac{p}{2}$

(b) $\tan\theta = \dfrac{PR}{QR} = \dfrac{p^2/4}{p-h} = \dfrac{p^2/4}{p-p/2} = \dfrac{p}{2}$

$\dfrac{d}{dt}(\tan\theta) = \dfrac{d}{dt}(\dfrac{1}{2}p) \Rightarrow \sec^2\theta\dfrac{d\theta}{dt} = \dfrac{1}{2}\dfrac{dp}{dt} \Rightarrow \dfrac{d\theta}{dt} = \dfrac{1}{2}(4)\cos^2\theta = 2\cos^2\theta$

When $p=2$, $QR = p-h = p-\dfrac{p}{2} = \dfrac{p}{2} = 1$ and $PR = \dfrac{p^2}{4} = \dfrac{2^2}{4} = 1$.

$PQ = \sqrt{1^2+1^2} = \sqrt{2} \Rightarrow \cos\theta = \dfrac{1}{\sqrt{2}}$.

$\dfrac{d\theta}{dt} = 2\cos^2\theta = 2(\dfrac{1}{\sqrt{2}})^2 = 1$

(c) Area of $\Delta PQR = A = \dfrac{1}{2}(p-h)(\dfrac{p^2}{4}) = \dfrac{1}{2}(p-\dfrac{p}{2})(\dfrac{p^2}{4}) = \dfrac{p^3}{16}$

$\dfrac{dA}{dt} = \dfrac{3p^2}{16}\cdot\dfrac{dp}{dt}$; $\left.\dfrac{dA}{dt}\right|_{p=2} = \dfrac{3(2)^2}{16}(4) = 3$

3. (a) $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{e^{\sin t}}{\sqrt{t^2+2}}$

$\left.\dfrac{dy}{dx}\right|_{t=1} = \dfrac{e^{\sin 1}}{\sqrt{1+2}} = 1.339$

$y + 2 = 1.339(x-1)$

(b) $y(\pi) - y(1) = \int_1^{\pi} e^{\sin t}\, dt$

$y(\pi) = y(1) + \int_1^{\pi} e^{\sin t}\, dt = -2 + 4.577 = 2.577$

(c) Speed $= \sqrt{\left[\dfrac{dx}{dt}\right]^2 + \left[\dfrac{dy}{dt}\right]^2} = \sqrt{(\sqrt{\pi^2+2})^2 + (e^{\sin\pi})^2} = \sqrt{\pi^2+2+1} = 3.587$

(d) Distance $= \int_a^b \sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2}\, dt = \int_0^{2\pi} \sqrt{(\sqrt{t^2+2})^2 + (e^{\sin t})^2}\, dt = 25.105$

SECTION II, Part B

4. (a) $f'(x) = \dfrac{4x\sqrt{f(x)}}{3}$; $f'(1) = \dfrac{4\sqrt{f(1)}}{3} = \dfrac{4\sqrt{4}}{3} = \dfrac{8}{3}$

$y - 4 = \dfrac{8}{3}(x-1)$ or $y = \dfrac{8}{3}x + \dfrac{4}{3}$

$f(1.1) \approx y(1.1) = \dfrac{8}{3}(1.1) + \dfrac{4}{3} = 4.267$

(b) $f''(x) = \dfrac{4}{3}\left[x \cdot \dfrac{f'(x)}{2\sqrt{f(x)}} + \sqrt{f(x)}\right]$

$f''(1) = \dfrac{4}{3}\left[\dfrac{f'(1)}{2\sqrt{f(1)}} + \sqrt{f(1)}\right] = \dfrac{4}{3}\left[\dfrac{8/3}{2\sqrt{4}} + \sqrt{4}\right] = \dfrac{32}{9}$

(c) $\dfrac{dy}{dx} = \dfrac{4x\sqrt{y}}{3} \Rightarrow \dfrac{dy}{\sqrt{y}} = \dfrac{4x}{3}dx \Rightarrow \int \dfrac{dy}{\sqrt{y}} = \int \dfrac{4x}{3}dx$

$\Rightarrow 2\sqrt{y} = \dfrac{2}{3}x^2 + C_1 \Rightarrow \sqrt{y} = \dfrac{1}{3}x^2 + \dfrac{1}{2}C \Rightarrow y = (\dfrac{1}{3}x^2 + \dfrac{1}{2}C)^2$

$y(1) = 4 \Rightarrow 4 = (\dfrac{1}{3} + \dfrac{1}{2}C)^2 \Rightarrow C = \dfrac{10}{3}$

Therefore, $y = (\dfrac{1}{3}x^2 + \dfrac{5}{3})^2$.

5. (a) $\frac{dP}{dt} = \frac{P}{2}\left(1 - \frac{P}{32}\right)$

For this logistic differential equation, the carrying capacity is 32.
Therefore, when $P(0) = 10$, $\lim_{t\to\infty} P(t) = 32$ and when $P(0) = 40$, $\lim_{t\to\infty} P(t) = 32$.

(b) The population is growing the fastest when P is half the carrying capacity.
Therefore, the population is growing the fastest when $P = 16$.

(c) The graph of P has a point of inflection when P is half the carrying capacity.

Slope $= \left.\frac{dP}{dt}\right|_{P=16} = \frac{16}{2}\left(1 - \frac{16}{32}\right) = 4$

(d) $\frac{dy}{dx} = \frac{y}{2}\left(1 - \frac{x}{32}\right) \Rightarrow \frac{dy}{y} = \frac{1}{2}\left(1 - \frac{x}{32}\right)dx = (\frac{1}{2} - \frac{x}{64})dx$

$\Rightarrow \ln|y| = \frac{x}{2} - \frac{x^2}{128} + C_1 \Rightarrow y = Ce^{\frac{x}{2} - \frac{x^2}{128}}$

$y(0) = \frac{1}{2} \Rightarrow C = \frac{1}{2}$. Therefore, $y = \frac{1}{2}e^{\frac{x}{2} - \frac{x^2}{128}}$.

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6. (a) $f(3) = 1$; $f'(3) = \frac{2!}{2^1} = 1$; $f''(3) = \frac{3!}{2^2}$; $f'''(3) = \frac{4!}{2^3}$

$$f(x) = 1 + (x-3) + \frac{3!}{2!2^2}(x-3)^2 + \frac{4!}{3!2^3}(x-3)^3 + \cdots + \frac{(n+1)!}{n!2^n}(x-3)^n + \cdots$$
$$= 1 + (x-3) + \frac{3}{2^2}(x-3)^2 + \frac{4}{2^3}(x-3)^3 + \cdots + \frac{n+1}{2^n}(x-3)^n + \cdots$$

(b) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+2)(x-3)^{n+1}}{2^{n+1}} \cdot \frac{2^n}{(n+1)(x-3)^n}\right| = \lim_{n\to\infty}\left|\frac{n+2}{n+1} \cdot \frac{1}{2}(x-3)\right|$

$= \frac{1}{2}|x-3| < 1$ when $|x-3| < 2$.

The radius of convergence is 2.

(c) $h'(x) = f(x)$

$$h(x) = \int f(x)\,dx = C + \int \left[1 + (x-3) + \frac{3}{2^2}(x-3)^2 + \cdots + \frac{n+1}{2^n}(x-3)^n + \cdots\right] dx$$
$$= C + \left[(x-3) + \frac{1}{2}(x-3)^2 + \frac{1}{2^2}(x-3)^3 + \cdots + \frac{1}{2^n}(x-3)^{n+1} + \cdots\right]$$

$h(3) = 2 \Rightarrow C = 2$

$$h(x) = 2 + (x-3) + \frac{1}{2}(x-3)^2 + \frac{1}{2^2}(x-3)^3 + \cdots + \frac{1}{2^n}(x-3)^{n+1} + \cdots$$

(d) The Taylor series for h is a geometric series with $r = (x-3)/2$. Since the geometric series only converges for $|r| = \left|\frac{x-3}{2}\right| < 1$, the Taylor series does not converge at $x = 1$.